An Algebraic Combinatorial Approach to the Abstract Syntax of Opetopic Structures

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Abstract

The starting point of the talk will be the identification of structure common to tree-like combinatorial objects, exemplifying the situation with abstract syntax trees (as used in formal languages) and with opetopes (as used in higher-dimensional algebra). The emerging mathematical structure will be then formalized in a categorical setting, unifying the algebraic aspects of the theory of abstract syntax of [2, 3] and the theory of opetopes of [6]. This realization conceptually allows one to transport viewpoints between these, now bridged, mathematical theories and I will explore it here in the direction of higher-dimensional algebra, giving an algebraic combinatorial framework for a generalisation of the slice construction of [1] for generating opetopes. The technical work will involve setting up a microcosm principle for near-semirings [5] and subsequently exploiting it in the cartesian closed bicategory of generalised species of structures of [4]. Connections to Homotopy Type Theory, (cartesian and symmetric monoidal) equational theories, lambda calculus, and algebraic combinatorics will be mentioned in passing.

References

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